Mathematical Models of Perception and Generation of Art Works by Dynamic Motions

Alexander Schubert, Katja Mombaur, and Joachim Funke

Abstract This paper presents a study on the role of dynamic motions in the creation and perception processes of action-art paintings. Although there is a lot of interest and some qualitative knowledge around, there are no quantitative models in the scientific computing sense about this process yet. To create such models and implement them on a robotic platform is the objective of our work. Therefore, we performed motion capture experiments with an artist and reconstructed the recorded motions by fitting the data to a rigid-body model of the artist's arm. A second model of a 6-DOF robotic platform is used to generate new motions by means of optimization and optimal control algorithms. Additionally, we present an image analysis framework that computes certain image characteristics related to aesthetic perception and a web tool that we developed to perform online sorting and cluster studies with participants. We present first results concerning motion reconstruction and perception studies and give an outlook to what will be the next steps towards an autonomous painting robotic platform.

1 Introduction

The cognitive processes of generating and perceiving abstract art are – in contrast to figurative art – mostly unknown. Within the process of perceiving representational art works, the effect of meaning is highly dominant. In abstract art, with the lack of this factor, the processes of perception are much more ambiguous, relying on a variety of more subtle qualities. In this work, we focus on the role of dynamic motions performed during the creation of an art work as one specific aspect that influences our perception and aesthetic experience.

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1.1 Action Paintings: Modern Art Works Created by Dynamic Motions

The term “action painting” was first used in the essay “The American Action Painters” by Harold Rosenberg in 1952 [1]. While the term “action painting” is commonly used in public, art historians sometimes also use the term “Gestural Abstraction”. Both terms emphasize the process of creating art, rather than the resulting art work, which reflects the key innovation that arose with this new form of painting in the 1940s to the 1960s. The artists often consider the physical act of painting itself as the essential aspect of the finished work. The most important representative of this movement is Jackson Pollock (1912–1958), who introduced this new style around 1946. Clearly, artists like Pollock do not think actively about dynamic motions performed by their bodies the way, mathematicians from the area of modeling and optimal control do. But from a mathematical and biomechanical point of view it is very exciting that one of the main changes they applied to their painting style in order to achieve their aim of addressing the subconscious mind has been a shift in the manner they carry out their motions during the creational process

1.2 Understanding the Perception and Generation of Art Works

Since humans possess many more degrees of freedom than needed to move a hand (or any end-effector that they might be using for painting, like brushes or pencils), the motions executed by an artist can be seen as a superposition of goal directed motions and implicit, unconscious motions. The former are carried out to direct his hand to the desired position, the latter are the result of some unconscious process defining a particular style of the motion. From a mathematical perspective, this can be seen as an implicitly solved optimal control problem with a certain cost function

Fig. 1 An action painting in the style of Jackson Pollock, painted by “JacksonBot”

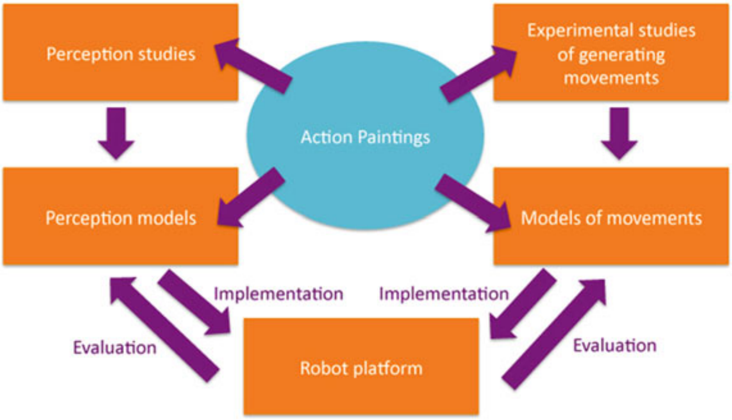

Fig. 2 Schematic overview of experimental and computational parts of study

relating to smoothness, jerk, stability or energy costs. The assumption that human motion can be described in this manner has been widely applied and verified, for example in human locomotion. For details, see [2] or [19].

When looking at action paintings, we note that this form of art generation is a very extreme form of this superposition model with a negligible goal-directed part. Therefore, it is a perfect basis to study the role of (unconscious) motion dynamics on a resulting art work.

The goal of our project is to use state-of-the-art tools from scientific computing to analyze the impact of motion dynamics both on the creational and perceptual side of action-painting art works. Figure 2 shows a schematic overview of the experimental and theoretical parts of our project. On the one hand, we perform perception studies, in which participants are shown different action paintings and then have to describe how they perceive these paintings. On the basis of these experiments, models for the perception of action paintings are established. On the other hand, we have conducted motion capture studies in which an artist generated action paintings. The painting process was recorded using several inertia sensors on the artist's arm and hand which provide both kinematic and dynamic data. On the basis of these recordings, we reconstructed and analyzed the artist's motion. Results from both approaches – on perception and on the generation of action art – will later be implemented on a robot for validation purposes. In this paper, we present some preliminary results on modeling, motion reconstruction as well as on perception studies and our image analysis framework.

1.3 Paper Outline

This paper is organized as follows: In Sect. 2, we will give an introduction to the current theory of art perception and an overview of the tools we developed for image analysis and online perception experiments. In Sect. 3, we first briefly discuss the mathematical background of our work by introducing optimal control problems and the direct multiple shooting method. Then, we describe the reconstruction of recorded motions from an artist using multibody dynamics and optimal control theory. Thereafter, we present our plan to create new motions for our robotic platform by solving an optimal control problem to compute the joint torques. Finally, in Sect. 4, we conclude our current findings and present the next steps in our project plan.

2 Modeling the Perception of Art Works

When we talk about models for art perception in this paper, we have to state that we do not want to create a new qualitative model for art perception but we want to find quantitative data that link the motion dynamics of the creation process to viewers' aesthetic experience when looking at the painting. Once we find this data, we aim to integrate it into existing perception models, possibly modifying or improving them. Our main goal is, however, to develop a simple mathematical model that allows our robotic platform to continuously monitor its painting process and to adapt its motion dynamics considering previously given goals.

2.1 Previous Work/State of the Art

The perception of art, especially abstract art, is still an area of ongoing investigations. Therefore, no generally accepted theory including all facets of art perception exists. There are, however, different theories that can explain different aspects of art perception. One example of a theory of art perception is the one presented by Leder et al. in [3] (see Fig. 3). In the past, resulting from an increasing interest in embodied cognition and embodied perception, there has been a stronger focus on the nature of human motion and its dynamics regarding neuroscience or rather neuroaesthetics as well as psychology and history of art. There are several results, showing that we perceive motion and actions with a strong involvement of those brain regions that are responsible for motion and action generation. These findings support the theory that the neural representations for action perception and action production are identical (see, e.g. [4]). The relation between perception and embodied action simulation also exists for static scenes (see, e.g. [5]) and ranges even to the degree, where the motion is implied only by a static result of this very motion. For example, Knoblich et al. showed in [6] that the observation of a static graph sign evokes in the brain a motor simulation of the gesture, which is required to produce this graph sign. Finally, in [7], D. Freedberg and V. Gallese proposed that this effect of reconstructing motions by embodied simulation mechanisms will also be found when looking at "art works that are characterized by the particular

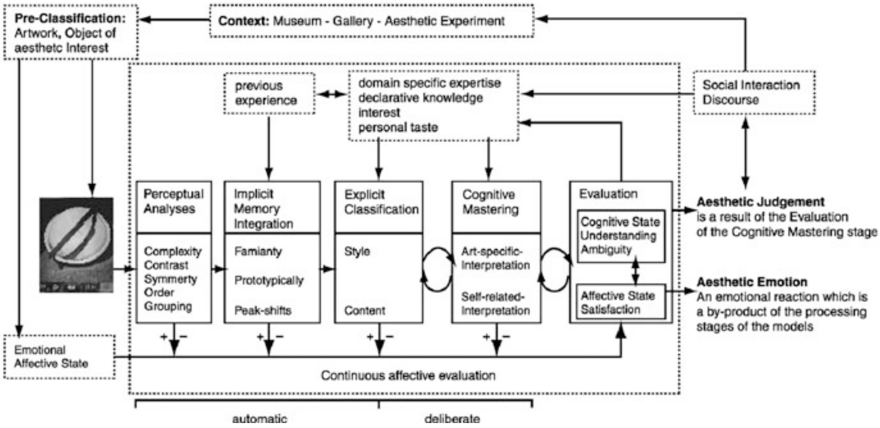

Fig. 3 Figure taken from Leder et al. [3]

gestural traces of the artist, as in Fontana and Pollock" – a conjecture that has first been observed empirically by Taylor et al. in [8].

2.2 Perception Experiments

This section describes our perception experiments which are performed using a web interface that we created for this purpose.

We performed two pre-studies to find out, whether human contemplators can distinguish robot paintings from human-made paintings and how they evaluate robot paintings created by the robot JacksonBot [17] using motions that are the result of an optimal control problem with different mathematical objective functions. In the first study, we showed nine paintings to 29 participants, most of whom were laymen in arts and only vaguely familiar with Jackson Pollock. Seven paintings were original art works by Jackson Pollock and two paintings were generated by the robot platform JacksonBot. We asked the participants to judge which of the paintings were original paintings by Pollock and which were not, but we intentionally did not inform them about the robotic background of the "fake" paintings. As might be expected, the original works by Pollock had a higher acceptance rate, but, very surprisingly, the difference between Pollock's and JacksonBot's paintings was not very high (2.74 ± 0.09 vs. 2.85 ± 0.76, on a scale of 1–5).

In the second study, the participants were shown ten paintings created solely by the robot platform, but with two different objective functions (maximizing and minimizing overall angular velocity in the robot arm) in the optimal control problem. The participants easily distinguished the two different painting styles.

After the pre-studies, we developed a more sophisticated web-based platform for further, more detailed investigations on this subject.

The first goal of our detailed perception experiments is to find out about the way, viewers judge action-art paintings regarding similarity. Therefore, we present a set of stimuli consisting of original action-art paintings by Pollock and other artists and added images, that were painted by our robot platform. Participants are then asked to perform different tasks with these stimuli.

The web-interface provides three different study types for perception analysis. In the first task, the viewers are presented three randomly chosen paintings and asked to arrange them on the screen according to their similarity. As a result, for every set of three paintings A, B, C, we obtain a measure $d_{ABC} = \frac{dist_{AB}}{dist_{BC}}$ for the similarity of two paintings in comparison with another pair of two paintings.

In the second task, people are basically asked to perform a standard sorting study, i.e. they are asked to combine similar paintings in groups and to give some information about their reasons for the chosen groups (Fig. 4). The results of this task are used to validate the information obtained by the previous one and, additionally, they are used to gain more information about the attributes and traits, people seem to use while grouping.

Finally, participants are shown images individually and are asked to judge them on different absolute scales. The results from this task are used to obtain an overall scaling for the first two tasks.

Once, we have obtained this information for a sufficient amount of robot paintings, we can use standard procedures from statistics like fuzzy cluster analysis or multidimenstional scaling to determine whether viewers differentiate between paintings created by different objective functions or rather whether they rate paintings created by the same objective function as similar. Additionally, we can link the given cluster descriptions to certain objective functions (e.g. paintings created by maximum jerk motions might be clustered together and be described as "aggressive" or "dynamic").

Fig. 4 Interface for web-based perception studies

2.3 Perception Models

As stated in Sect. 2, we want to develop a model that allows our robotic platform to monitor its painting process using a camera system and – based on an evaluation of its current status – to change its movement according to predefined goals. Therefore, we developed an image analysis software tool based on OpenCV for details, see [9] that uses a variety of different filters and image processing tools that are related to aesthetic experience. For an overview on the software, see [10]. To give only one example, Taylor et al. showed in [11] that fractal-like properties of art works might be of interest, particularly when looking at action-art paintings. We address the question of fractal-like properties by computing two values: the fractal dimension D using the "box counting" method and the Fourier power spectrum using FFT. The fractal dimension is calculated by overlapping the given image with a continuously refining two-dimensional grid of width ϵ. If $N(\varepsilon)$ is the number of "boxes" that cover a part of the object of interest, the fractal dimension is given by:

$$D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log \frac{1}{\varepsilon}} \tag{1}$$

By linking these low-level image features to the viewer's judgements described in the previous paragraph, the robot will be able to predict the most likely judgement of a viewer and to adapt its movement accordingly.

3 Modeling the Generation of Art Works by Dynamic Motions

As mentioned in Sects. 1.1 and 1.2, the generation of action paintings uses motions that arise from the subconscious of the artists. Therefore, we cannot try to generate similar motions by traditional path planning. Instead, we apply our approach of generating motions as the result of an optimal control problem, which is much more suited to address this type of motions.

3.1 Mathematical Background

To perform mathematical computations on motion dynamics, we first need to create models of a human and the robot arm. In this case, by "model", we mean a physical multi-body model consisting of rigid bodies which are connected by different types of joints (prismatic or revolute). Depending on the number of bodies and joints, we end up with an certain number of degrees of freedom and a set of generalized variables q (coordinates), $\dot{q}$ (velocities), $\ddot{q}$ (accelerations), and τ (joint torques). Given such a model, we can fully describe its dynamics by means of

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau \tag{2}$$

where $M(q)$ is the joint space inertia matrix and $N(q,\dot{q})$ contains the generalized non-linear effects. Once we have such a model, we can formulate an optimal control problem using $x = [q,\dot{q}]^T$ as states and $u = \tau$ as controls. The OCP can be written in a general form as:

$$\min_{x,u,T_1} \int_{T_0}^{T_1} L(t,x(t),u(t),p)dt + \Phi_M(T_1,x(T_1)) \tag{3}$$

subject to:

$$\dot{x} = f(t,x(t),u(t),p) \tag{4}$$

$$g(x(t),u(t),p) \geq 0 \tag{5}$$

$$r_{T_0}(x(T_0),p) + r_{T_1}(x(T_1),p) = 0 \tag{6}$$

where p contains several model parameters which in our case are fixed and g contains constraints like joint and torque limitations. Note, that all the dynamic computation from our model is included in the RHS of diff.eq. (4), The objective function is given by the sum of the Lagrange term $\int_{T_0}^{T_1} L(t,x(t),u(t),p)dt$ and the Mayer term $\Phi_M(T_1,x(T_1))$. The former is used to address objectives that have to be evaluated over the whole time horizon (such as minimizing jerk), the latter is used to address objectives that only need to be evaluated at the end of the time horizon (such as overall time). In our case, we will often only use the Lagrange term. For details about the specific problems we used, see Sects. 3.3 and 3.4.

To solve such a problem numerically, we apply a direct multiple shooting method which was developed by Bock and Plitt [12] and is implemented in the software package MUSCOD-II, which is maintained and developed further at IWR. It discretizes the continuous formulation of our optimal control problem by dividing the time horizon in several so-called multiple shooting intervals I_j. This discretization is used both for controls and states, the latter are parameterized as starting values s_j for an initial value problem on each multiple shooting interval I_j. The controls are given by simple base functions $\bar{u}|_{I_j}$ (e.g. piece-wise constant, piece-wise linear or spline functions) for each interval. Additional continuity conditions

$$x(t_{j+1}, s_j, \bar{u}|_{I_j}) - s_{j+1} = 0$$

are added for each multiple-shooting-node to ensure a continuous solution. Further discretization of the constraints and objective function leads to a nonlinear optimization problem:

$$\min_{y} F(y) \tag{7}$$

subject to:

$$g(y) \geq 0 \tag{8}$$

$$h(y) = 0 \tag{9}$$

where y contains the variables s_j, T_1 and the parameters describing the control base functions $\bar{u}|_{I_j}$. This problem is then solved by using a specially tailored sequential quadratic programming (SQP) method. For a more detailed description of the algorithm, see [12, 13]. Regarding dynamics computation, we use the Rigid Body Dynamics Library (RBDL) [14] which is an highly efficient C++ library for forward and inverse rigid body dynamics and includes all major algorithms like the articulated body algorithm and a recursive Newton-Euler algorithm.

3.2 Previous Work/State of the Art

Optimization and optimal control techniques are very powerful tools that can be applied concerning many aspects of our research. In this specific case, we use optimization methods to compute the full trajectory of our robotic platform. Our basic approach is that humans are unwittingly applying optimization in different areas like motion control or complex problem solving. As mentioned in Sect. 1.2, this approach of characterizing human motions as solution of an optimal control problem has been successfully applied in several areas, particularly in walking and running motions (see [2, 15]), but also (very recently) regarding emotional body language during human walking (see [16]). Concerning the application of our approach on painting motions, a first proof of concept has been given by our previous robotic platform "JacksonBot". Even though with "JacksonBot", the optimization was purely kinematic with no respect to motion dynamics, paintings created using different optimality conditions were clearly distinguished by viewers (see [17]).

3.3 Experiments with Artists

In order to study the way, real human artists move during action-painting, we performed motion-capture studies. We started with several experiments where we recorded the motion of a collaborating artist and plan to redo the same experiments with other artists for validation purposes. We used three inertia sensors to record dynamic data $D_{capture}$ for each of the three segments of the artist's arm (hand, lower arm, upper arm). To fit this data to our 9 DOF model of a human arm that is based on data from deLeva [18], we formulated an optimal control problem which generates the motion $x(t) = [q(t), \dot{q}(t)]^T$ and the controls $u(t) = \tau(t)$ that best fit the captured data with respect to the model dynamics f.

$$\min_{\alpha} \sum_{i} ||D^*(x(t_i;\alpha)) - D_{mocap}(t_i)||_2^2, \tag{10}$$

$D^*(x(t;\alpha))$ resulting from a solution of

$$\min_{x,u} \int_{T_0}^{T_1} \left[\sum_{i=1}^{n} \alpha_i L_i(t, x(t), u(t), p) \right] dt \tag{11}$$

subject to:

$$\dot{x}(t) = f(t, x(t), u(t), p) \tag{12}$$

$$r_{T_0}(x(T_0), p) + r_{T_1}(x(T_1), p) = 0 \tag{13}$$

$$g(x(t), u(t), p) \geq 0 \tag{14}$$

The constraints in this case are given by the limited angles of the human arm joints and torque limitations of the arm muscles. Figure 5 shows the computed states and the fit quality of the acceleration data for a very dynamic, jerky motion. Note that for this type of motion, the fact that the angle values are approaching the joint limitations is plausible.

3.4 Motion Generation for Robot Platform by Means of Optimal Control

To generate new motions for our robotic platform (a 6-DOF-KUKA arm) we created a 6-DOF rigid-body-model of the arm. We now can compute end-effector trajectories as results of optimal control problems with different objective functions. The mathematical problem is described and solved using the optimal control code MUSCOD-II as it has been described in Sect. 3.1. In this case, we include all limitations of our KUKA arm using the inequality constraints $g(x(t), u(t), p) \geq 0$ and choose from a set of different objective functions L derived either from our motion capture experiments or motivated from physical extremes (e.g. maximizing the torque or minimizing the variance of the angular velocities in all joints).

The paintings created by the robot based on (a superposition of) these objective functions will be added to the paintings already present in the framework of our perception studies. This has two major advantages compared to human-created paintings: First, we know the exact details about the underlying motion dynamics

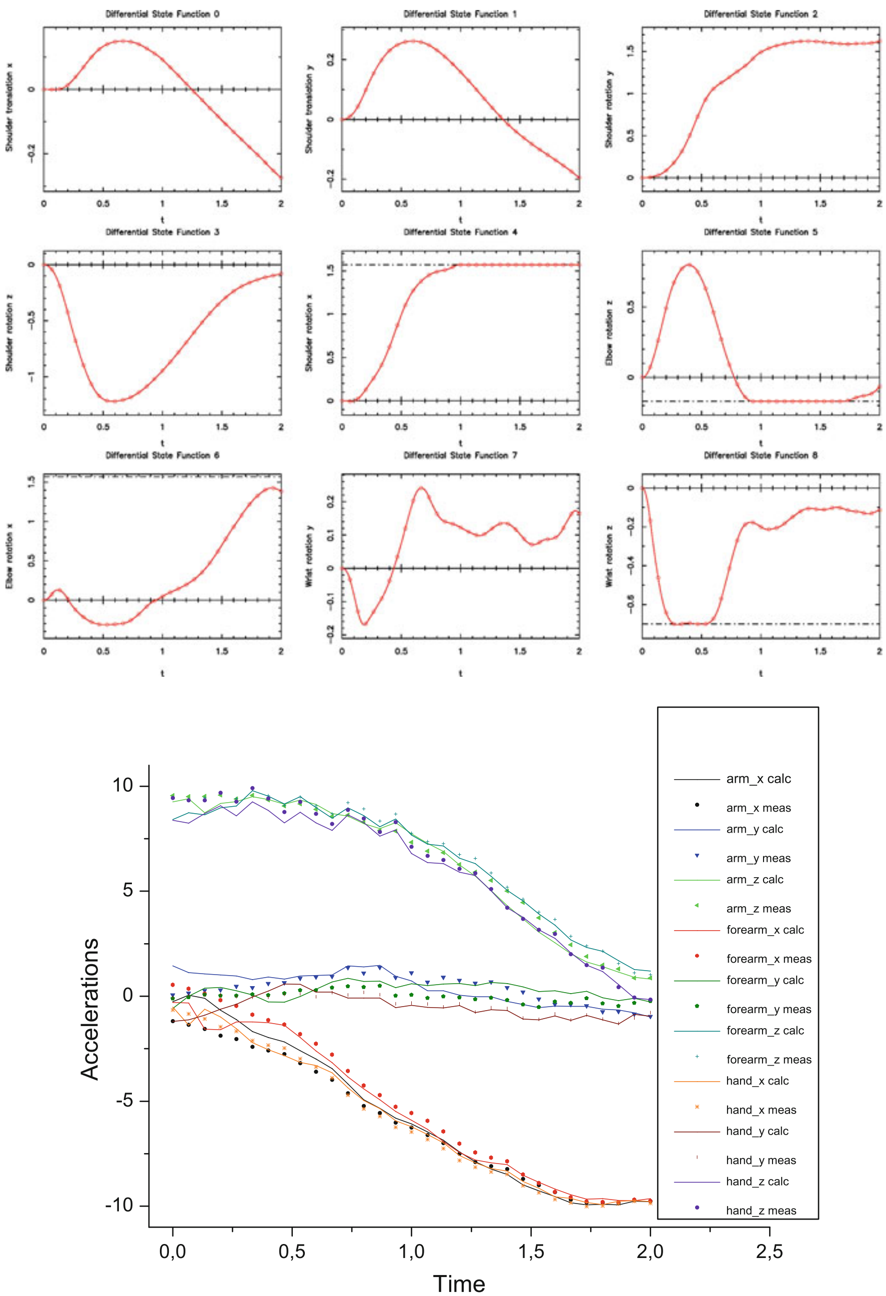

Fig. 5 Computed trajectories for joint angles (*above*) and comparison of computed (*dots*) and measured (*lines*) accelerations (*below*)

and can therefore derive correlations more easy. Second, we can easily create images specifically suited to an area of interest in our perception study.

4 Summary

An overview of our approach to investigate the influence of dynamic motions on modern art works was presented. We successfully reconstructed artist's motions from dynamic motion-capture data using a rigid-body model of the artist's arm. We described the advantages of our optimal control approach to this specific type of human motions and portrayed the combination of several tools for perception studies and image analysis with a robotic platform in order to uncover the subconscious nature of action-painting motions. In the next step, we will use the motion capture data obtained from experiments with our collaborating artist not only to reconstruct the motion, but to use an inverse optimal control approach (like successfully used in a similar case by Mombaur et al. in [19]) to retrieve the underlying objective functions of these motions. To do so, we will use an efficient direct all-at-once approach as presented by Hatz et al. in [20]. We will link these objectives both to low-level image features detected by our image analysis framework and viewers' judgements derived from our online-tool. That way, we aim to build a database containing all this information as a foundation to create a feedback for the robot painting process.

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